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Quaternion-Based Adaptive Attitude Tracking Controller Without Velocity Measurements

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The main problem addressed is the quaternion-based, attitude tracking control of rigid spacecraft without angular velocity measurements and in the presence of an unknown inertia matrix. As a stepping stone, an adaptive, full-state feedback controller is designed that compensates for parametric uncertainty while ensuring asymptotic attitude tracking errors. The adaptive, full-state feedback controller is then redesigned such that the need for angular velocity measurements is eliminated. The proposed adaptive, output feedback controller ensures asymptotic attitude tracking. A four-parameter representation is used of the spacecraft attitude that does not exhibit singular orientations as in the case of the previous three-parameter representation-based results. To the best of our knowledge, this represents the first solution to the adaptive, output feedback, attitude tracking control problem for the quaternion representation. Simulation results are included to illustrate the performance of the proposed output feedback control strategy.

I. Introduction

THE attitude control of rigid bodies has important applications ranging from rigid aircraft and spacecraft systems to coordinated robot manipulators (see Ref. 1 for a literature review of the many different types of applications). For example, rigid spacecraft applications in particular, for example, satellite surveillance and communication, often have need of highly accurate slewing and/or pointing maneuvers that require the spacecraft to rotate along a relatively large-angle amplitude trajectory. As noted in Ref. 2, these requirements necessitate the use of a nonlinear dynamic spacecraft model for control system synthesis. The control problem is further complicated by the uncertainty of the spacecraft mass and inertia properties due to fuel consumption, payload variation, appendage deployment, etc.

The attitude motion of a rigid body is basically represented by a set of two equations[1,3]: 1) Euler's dynamic equation, which describes the time evolution of the angular velocity vector, and 2) the kinematic equation, which relates the time derivatives of the orientation angles to the angular velocity vector. Several kinematic parameterizations[4] exist to represent the orientation angles, including singular, three-parameter representations, that is, the Euler angles, Gibbs vector, Cayley–Rodrigues parameters, and modified Rodrigues parameters, and the nonsingular, four-parameter representation given by the unit quaternion, that is, the Euler parameters. Whereas the three-parameter representations always exhibit singular orientations (i.e., the Jacobian matrix in the kinematic equation is singular for some orientations, the unit quaternion globally represents the spacecraft attitude without singularities; however, an additional constraint equation is introduced through the use of the four-parameter representation.

Several solutions to the attitude control problem have been presented in the literature since the early 1970s.[5] See Ref. 1 for a comprehensive literature review of earlier work. In Ref. 1, the authors presented a general attitude control design framework which includes proportional-derivative, model based, and adaptive setpoint controllers. Adaptive tracking control schemes based on three-parameter, kinematic representations were presented in Refs. 6 and 7 to compensate for the unknown, spacecraft inertia matrix. In Ref. 2, an adaptive attitude tracking controller based on the unit quaternion was proposed that identified the inertia matrix via periodic command signals. The work in Ref. 2 was later applied to the angular velocity tracking problem in Ref. 8. An $\mathcal{H}_\infty$-suboptimal state feedback controller was developed for the quaternion representation in Ref. 9. In Ref. 10, the authors designed an inverse optimal control law for attitude regulation using the backstepping method for a three-parameter representation. A passivity-based approach was presented in Ref. 11 that used quaternions to develop an adaptive tracking controller for rigid spacecraft. Recently, in Ref. 12, a variable structure tracking controller was presented using quaternions in the presence of spacecraft inertia uncertainties and external disturbances.

A typical feature in all of the mentioned attitude control schemes is that angular velocity measurements are required. Unfortunately, this requirement is not always satisfied in reality. Thus, a common practice is to approximate the angular velocity signal through an ad hoc numerical differentiation of the attitude angles and directly use this surrogate signal for control design with no guarantee of closed-loop stability. With this in mind, an angular velocity observer was developed in Ref. 13 for the quaternion representation; however, the observer was based on an unproven separation principle argument. In Ref. 14, a passivity approach was used to develop an asymptotically stabilizing setpoint controller that eliminated velocity measurements via the filtering of the unit quaternion. The passivity-based, velocity-free setpoint controller of Ref. 14 was later applied to the simpler, three-parameter problem in Ref. 15. Recently, in Ref. 16 the results of Refs. 14 and 15 were extended to the tracking problem using the modified Rodrigues parameters; however, the proposed control law requires exact knowledge of the spacecraft inertia. This constraint was overcome in Ref. 17 via an adaptive output feedback controller, which compensated for inertia-related uncertainty. Two quaternion-based, output feedback controllers using a model-based observer and a lead filter were proposed in Ref. 18

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to estimate the angular velocity; however, exact knowledge of the spacecraft inertia is required by the control law.

In this paper, we provide an adaptive control solution to the quaternion-based, attitude tracking control problem that eliminates angular velocity measurements and compensates for parametric uncertainty. Specifically, we first apply a novel transformation to the open-loop, quaternion tracking error dynamics. The transformed tracking error dynamics are then used to design a new adaptive, full-state feedback controller that compensates for uncertainties in the inertia matrix. A nonstandard, Lyapunov-like function, which exploits the quaternion constraint equation, is used to prove asymptotic attitude tracking. To achieve the goal of elimination of velocity measurements, we then exploit the structure of the adaptive, full-state feedback controller and its corresponding stability argument. Specifically, we utilize a filter, whose structure is motivated by the Lyapunov-like stability analysis, to generate a velocity-related signal from attitude measurements. The proposed output feedback controller is shown to guarantee asymptotic attitude tracking. To the best of our knowledge, this represents the first solution to the adaptive, output feedback, attitude tracking control problem for the quaternion representation (note that the adaptive output feedback result of Ref. 17 was done for the simpler modified Rodrigues parameter case). Note that the solution to this problem is nontrivial due to the nonsquare, that is, 4×3, nature of the Jacobian matrix in the kinematic equation. As a result, a judicious error system development, control design, and Lyapunov-like stability analysis, which make appropriate use of the quaternion constraint equation, are crucial to the solution of the problem.

The paper is organized as follows. Section II contains the derivation of the spacecraft model. The adaptive, full-state feedback controller is presented in Sec. III, whereas the output feedback controller is developed in Sec. IV. Simulation results are provided in Sec. V. Section VI presents some concluding remarks.

II. Model Formulation

A. Spacecraft Dynamics

We consider the problem of a rigid spacecraft with actuators that provide body-fixed torques about a body-fixed reference frame $\mathcal{B}$ located at some point on the spacecraft.[2] The body-fixed torques can be applied to each axis by a pair of equal but opposite forces that act in a direction perpendicular to the line joining the actuators. We then translate this body-fixed frame $\mathcal{B}$ to another body-fixed frame $\mathcal{F}$ with the same orientation, but located at the center of mass of the spacecraft. The dynamic model for the described rigid spacecraft can be expressed as follows[3]:

$$J\dot{\omega} = -\omega^{\times} J\omega + u \tag{1}$$

$$\dot{q} = \tfrac{1}{2}\left(q^{\times}\omega + q_0\omega\right) \tag{2}$$

$$\dot{q}_0 = -\tfrac{1}{2}q^T\omega \tag{3}$$

where $J \in \Re^{3\times 3}$ is the constant, positive-definite, symmetric inertia matrix, $\omega(t) \in \Re^3$ is the angular velocity of the body-fixed reference frame $\mathcal{F}$ with respect to an inertial reference frame $\mathcal{I}$, $u(t) \in \Re^3$ is a vector of control torques, and the notation $\zeta^{\times}$, $\forall\zeta = [\zeta_1 \ \ \zeta_2 \ \ \zeta_3]^T$, denotes the following skew-symmetric matrix:

$$\zeta^{\times} \triangleq \begin{bmatrix} 0 & -\zeta_3 & \zeta_2 \\ \zeta_3 & 0 & -\zeta_1 \\ -\zeta_2 & \zeta_1 & 0 \end{bmatrix} \tag{4}$$

In Eqs. (2) and (3), $\boldsymbol{q}(t) \triangleq \{q_0(t), q(t)\} \in \Re \times \Re^3$ represents the unit quaternion[3] describing the orientation of the body-fixed frame $\mathcal{F}$ with respect to the inertial frame $\mathcal{I}$, which are subject to the constraint

$$q^Tq + q_0^2 = 1 \tag{5}$$

The rotation matrix that brings $\mathcal{I}$ onto $\mathcal{F}$, denoted by $R(q, q_0) \in SO(3)$, is defined as follows:

$$R \triangleq \left(q_0^2 - q^Tq\right)I_3 + 2qq^T - 2q_0q^{\times} \tag{6}$$

where I_3 is the 3×3 identity matrix, and the angular velocity of $\mathcal{F}$ with respect to $\mathcal{I}$ expressed in $\mathcal{F}$, denoted by $\omega(t)$, can be computed from Eqs. (2) and (3) as follows:

$$\omega = 2(q_0\dot{q} - q\dot{q}_0) - 2q^{\times}\dot{q} \tag{7}$$

B. Open-Loop Tracking Error System Development

Similar to Ref. 2, we assume that the desired attitude of the spacecraft can be described by a desired, body-fixed reference frame $\mathcal{F}_d$ whose orientation with respect to the inertial frame $\mathcal{I}$ is specified by the desired unit quaternion $\boldsymbol{q}_d(t) \triangleq \{q_{0d}(t), q_d(t)\} \in \Re \times \Re^3$ that is constructed to satisfy

$$q_d^Tq_d + q_{0d}^2 = 1 \tag{8}$$

The corresponding rotation matrix, denoted by $R_d(q_d, q_{0d}) \in SO(3)$, that brings $\mathcal{I}$ onto $\mathcal{F}_d$ is then defined as follows:

$$R_d \triangleq \left(q_{0d}^2 - q_d^Tq_d\right)I_3 + 2q_dq_d^T - 2q_{0d}q_d^{\times} \tag{9}$$

The desired quaternion is related to the desired angular velocity of $\mathcal{F}_d$ with respect to $\mathcal{I}$ expressed in $\mathcal{F}_d$, denoted by $\omega_d(t) \in \Re^3$, through the following dynamic equations:

$$\dot{q}_d = \tfrac{1}{2}\left(q_d^{\times}\omega_d + q_{0d}\omega_d\right) \tag{10}$$

$$\dot{q}_{0d} = -\tfrac{1}{2}q_d^T\omega_d \tag{11}$$

Note that Eqs. (10) and (11) can be used to compute explicitly an expression for ω_d as

$$\omega_d = 2(q_{0d}\dot{q}_d - q_d\dot{q}_{0d}) - 2q_d^{\times}\dot{q}_d \tag{12}$$

To quantify the mismatch between the actual and desired spacecraft attitudes, we define the rotation matrix $\tilde{R}(e, e_0) \in SO(3)$ that brings $\mathcal{F}_d$ onto $\mathcal{F}$ as follows:

$$\tilde{R} \triangleq RR_d^T = \left(e_0^2 - e^Te\right)I_3 + 2ee^T - 2e_0e^{\times} \tag{13}$$

where $R(q, q_0)$ and $R_d(q_d, q_{0d})$ were defined in Eqs. (6) and (9), respectively, and the quaternion tracking error $\boldsymbol{e}(t) \triangleq \{e_0(t), e(t)\} \in \Re \times \Re^3$ is defined as

$$e_0 \triangleq q_0q_{0d} + q^Tq_d \tag{14}$$

$$e \triangleq q_{0d}q - q_0q_d + q^{\times}q_d \tag{15}$$

Note that, based on the definition given by Eq. (13), the attitude control objective can be stated as follows:

$$\lim_{t\to\infty} \tilde{R}[e(t), e_0(t)] = I_3 \tag{16}$$

Based on the preceding tracking error formulation, we define the angular velocity of $\mathcal{F}$ with respect to $\mathcal{F}_d$ expressed in $\mathcal{F}$, denoted by $\tilde{\omega}(t) \in \Re^3$, as follows:

$$\tilde{\omega} \triangleq \omega - \tilde{R}\omega_d \tag{17}$$

We can now use Eqs. (1–3), (10), (11), (14), (15), and (17) to compute the open-loop tracking error dynamics as follows:

$$J\dot{\tilde{\omega}} = -(\tilde{\omega} + \tilde{R}\omega_d)^{\times}J(\tilde{\omega} + \tilde{R}\omega_d) + J\left(\tilde{\omega}^{\times}\tilde{R}\omega_d - \tilde{R}\dot{\omega}_d\right) + u \tag{18}$$

$$\dot{e} = \tfrac{1}{2}\left(e^{\times} + e_0I_3\right)\tilde{\omega} \tag{19}$$

$$\dot{e}_0 = -\tfrac{1}{2}e^T\tilde{\omega} \tag{20}$$

where we have used the fact that $\dot{\tilde{R}} = -\tilde{\omega}^{\times}\tilde{R}$ in Eq. (18).

Remark 1: We will assume that $q_{0d}(t)$ and $q_d(t)$, and their first three time derivatives are bounded for all time. Note that this assumption ensures that $\omega_d(t)$ of Eq. (12) and its first two time derivatives are bounded for all time.

Remark 2: The relations given in Eqs. (14) and (15) can be explicitly calculated via quaternion algebra by noticing that the quaternion equivalent of Eq. (13) is the quaternion product (see Ref. 19 and Theorem 5.3 of Ref. 20).

$$\boldsymbol{e} = \boldsymbol{q}_d^* \boldsymbol{q} \tag{21}$$

where the unit quaternions $\boldsymbol{e}$ and $\boldsymbol{q}$ were defined earlier, and $\boldsymbol{q}_d^*(t) \triangleq \{q_{0d}(t),\ -q_d(t)\} \in \Re \times \Re^3$ is the unit quaternion representing the rotation matrix R_d^T.

Remark 3: After utilizing Eqs. (5), (8), (14), and (15), it is not difficult to show that the quaternion tracking error variables satisfy the following constraint:

$$e^T e + e_0^2 = 1 \tag{22}$$

Based on the constraint given by Eq. (22), we can see that

$$0 \leq \|e(t)\| \leq 1 \qquad 0 \leq |e_0(t)| \leq 1 \tag{23}$$

for all time, where $\|\cdot\|$ represents the standard Euclidean norm. It is also easy to see from Eq. (22) that

$$\text{if} \quad \lim_{t\to\infty} e(t) = 0, \ \text{then} \ \lim_{t\to\infty} |e_0(t)| = 1 \tag{24}$$

and, hence, we can see from Eq. (13) that if $\lim_{t\to\infty} e(t) = 0$ then the control objective defined by Eq. (16) will be achieved.

C. Transformed Open-Loop Tracking Error System

To express the open-loop tracking error dynamics given in Eqs. (18–20) in a more convenient manner, we first rewrite Eq. (19) as follows:

$$\dot{e} = \tfrac{1}{2} T \tilde{\omega} \tag{25}$$

where the Jacobian-type matrix $T(e, e_0) \in \Re^{3\times 3}$ is defined as follows:

$$T \triangleq e^\times + e_0 I \tag{26}$$

After taking the time derivative of Eq. (25) and premultiplying both sides of the resulting expression by $T^{-T} J T^{-1}$, we obtain the following:

$$J^* \ddot{e} = \tfrac{1}{2} J^* \dot{T} \tilde{\omega} + \tfrac{1}{2} P^T J \dot{\tilde{\omega}} \tag{27}$$

where $J^*(e, e_0) \in \Re^{3\times 3}$ is an auxiliary matrix defined as

$$J^* \triangleq P^T J P \tag{28}$$

and $P(e, e_0) \in \Re^{3\times 3}$ is defined as

$$P \triangleq T^{-1} \tag{29}$$

After substituting Eq. (18) into the right-hand side of Eq. (27), we can obtain the following expression for the open-loop tracking error dynamics (see Appendix A for details):

$$J^*(e, e_0)\ddot{e} + C^*(e, e_0, \dot{e})\dot{e} + N^*(e, e_0, \dot{e}, \omega_d, \dot{\omega}_d) = u^* \tag{30}$$

where the new control input $u^*(t) \in \Re^3$ is defined as

$$u^* \triangleq \tfrac{1}{2} P^T u \tag{31}$$

and the auxiliary dynamic terms $C^*(e, e_0, \dot{e}) \in \Re^{3\times 3}$ and $N^*(e, e_0, \dot{e}, \omega_d, \dot{\omega}_d) \in \Re^3$ are defined as follows:

$$C^* \triangleq -J^* \dot{P}^{-1} P - 2P^T (JP\dot{e})^\times P \tag{32}$$

$$N^* \triangleq P^T\left[(P\dot{e})^\times J\tilde{R}\omega_d\right] + P^T\left[(\tilde{R}\omega_d)^\times JP\dot{e}\right] + \tfrac{1}{2} P^T\left[(\tilde{R}\omega_d)^\times J\tilde{R}\omega_d\right] - \tfrac{1}{2} P^T J\left[(2P\dot{e})^\times \tilde{R}\omega_d - \tilde{R}\dot{\omega}_d\right] \tag{33}$$

The dynamic model given in Eq. (30) is characterized by the following two properties that will be utilized in the subsequent control development and analysis.

Property 1: The inertia and centripetal–Coriolis matrices satisfy the following skew-symmetric relationship (see Appendix B):

$$\xi^T\left(\tfrac{1}{2}\dot{J}^* - C^*\right)\xi = 0 \qquad \forall \xi \in \Re^3 \tag{34}$$

Property 2: The inertia matrix can be lower and upper bounded as follows:

$$j_1\|\xi\|^2 \leq \xi^T J \xi \leq j_2\|\xi\|^2 \qquad \forall \xi \in \Re^n \tag{35}$$

where j_1 and $j_2 \in \Re$ are some positive constants.

Remark 4: To ensure that $T(e, e_0)$ defined in Eq. (26) is invertible, it is a straightforward matter to show that we must guarantee that

$$\det(T) = e_0(t) \neq 0, \qquad \forall t \in [0, \infty) \tag{36}$$

To ensure that Eq. (36) remains valid, we will require that the initial conditions be restricted such that $e_0(0) \neq 0$, and that the subsequent control strategies be designed to guarantee that $e_0(t) \neq 0$ for all time. With regard to the restriction on the initial conditions, it is easy to see from Eqs. (24) and (14) that the desired trajectory can always be initialized to guarantee that $e_0(0) \neq 0$; hence, the initial conditions restriction is actually a very mild restriction on the desired trajectory signals.

III. Adaptive Full-State Feedback Control Development

In this section, our control objective is to design an adaptive attitude controller for the open-loop tracking error dynamics given by Eq. (30) under the constraint that the spacecraft inertia matrix J is unknown. To quantify the parameteric mismatch, we define the parameter estimation error, $\tilde{\theta}(t) \in \Re^6$, as follows:

$$\tilde{\theta}(t) \triangleq \theta - \hat{\theta}(t) \tag{37}$$

where $\theta \in \Re^6$ is a constant, unknown vector of inertia parameters defined as follows:

$$\theta \triangleq [J_{11} \quad J_{12} \quad J_{13} \quad J_{22} \quad J_{23} \quad J_{33}]^T \tag{38}$$

with J_{ij} being the elements of J and $\hat{\theta}(t) \in \Re^6$ being a dynamic estimate for θ which is yet to be defined. To facilitate the controller design, we also define the filtered tracking error, denoted by $r(t) \in \Re^3$, as follows:

$$r \triangleq \dot{e} + \alpha e \tag{39}$$

where $e(t)$ and $\dot{e}(t)$ were defined in Eqs. (15) and (19), respectively, and $\alpha \in \Re^{3\times 3}$ is a constant, positive-definite, diagonal, control gain matrix.

A. Control Torque Input Design

Based on the open-loop tracking error system given by Eq. (30) and the subsequent stability analysis, we design the control input $u^*(t)$ as follows:

$$u^* = -Y(e, e_0, \dot{e}, \omega_d, \dot{\omega}_d)\hat{\theta} - Kr - e/(1 - e^T e)^2 \tag{40}$$

where $Y(e, e_0, \dot{e}, \omega_d, \dot{\omega}_d) \in \Re^{3\times 6}$ is a known regression matrix constructed according to the following parameterization [we note that Eq. (41) is linear in the elements of J because of the structure of Eqs. (28), (32), and (33)]:

$$Y(\cdot)\theta = J^*\alpha\dot{e} + C^*\alpha e - N^* \tag{41}$$

and $\boldsymbol{\theta}$ was defined in Eq. (38), $K \in \Re^{3\times 3}$ is a constant, positive-definite, diagonal, control gain matrix, $\hat{\theta}(t)$ is generated via the following dynamic update law:

$$\dot{\hat{\theta}} = \Gamma Y^T(\cdot) r \tag{42}$$

and $\Gamma \in \Re^{6\times 6}$ is a constant, positive-definite, diagonal, adaptation gain matrix. To develop the closed-loop tracking error system, we take the time derivative of Eq. (39) and then premultiply both sides of the resulting equation by J^* to obtain the following expression:

$$J^*\dot{r} = J^*\ddot{e} + J^*\alpha\dot{e} \tag{43}$$

After substituting Eq. (30) into Eq. (43), we obtain

$$J^*\dot{r} = -C^*r + Y\theta + u^* \tag{44}$$

where Eqs. (39) and (41) were utilized. After substituting Eq. (40) for $u^*(t)$, we obtain the final expression for the closed-loop tracking error system

$$J^*\dot{r} = -C^*r + Y\tilde{\theta} - Kr - e/(1 - e^T e)^2 \tag{45}$$

where $\tilde{\theta}(t)$ was defined in Eq. (37).

B. Stability Analysis

Theorem 1: Given the closed-loop dynamics given in Eqs. (39) and (45), the adaptive controller of Eqs. (40) and (42) ensures asymptotic attitude tracking in the sense that

$$\lim_{t\to\infty} e(t) = 0, \qquad \lim_{t\to\infty} \tilde{\omega}(t) = 0 \tag{46}$$

provided that the initial conditions are selected such that

$$e_0(0) \neq 0 \tag{47}$$

Proof: To prove Theorem 1, we define the nonnegative function:

$$V(t) \triangleq \tfrac{1}{2}[e^T e/(1 - e^T e)] + \tfrac{1}{2}y^T J y + \tfrac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \tag{48}$$

where $y(t) \in \Re^3$ is defined as

$$y \triangleq Pr \tag{49}$$

and $P(e, e_0)$ was defined in Eq. (29). After taking the time derivative of Eq. (48) and then making the appropriate substitutions from Eqs. (28), (42), (45), and (49), we obtain

$$\dot{V}(t) = \frac{e^T\dot{e} - e^T e e^T\dot{e} + e^T e e^T\dot{e}}{(1 - e^T e)^2} - r^T C^* r + r^T Y\tilde{\theta} - r^T K r$$
$$- \frac{r^T e}{(1 - e^T e)^2} + \frac{1}{2}r^T\dot{J}^* r - \tilde{\theta}^T Y^T(\cdot) r \tag{50}$$

After substituting Eq. (39) into Eq. (50) for $\dot{e}(t)$ and then canceling common terms, we have that

$$\dot{V}(t) = -e^T\alpha e/(1 - e^T e)^2 - r^T K r \tag{51}$$

where Property 1 has been utilized.

Given Eqs. (22), (23), and (47), it is clear that $\|e(0)\| < 1$. From Eqs. (48) and (51), it is straightforward to see that

$$0 \leq V(t) \leq V(0) < \infty \tag{52}$$

hence, we can see from Eq. (48) that $y(t), \hat{\theta}(t) \in \mathcal{L}_\infty$, and $\|e(t)\| < 1$ for all time. Because $\|e(t)\| < 1$ for all time, we can conclude from Eq. (22) that $e_0(t) \neq 0$ for all time; hence, we know from Eq. (36) that P defined in Eq. (29) has full rank for all time. Because $y(t) \in \mathcal{L}_\infty$ and P has full rank for all time, we can use Eq. (49) to show that $r(t) \in \mathcal{L}_\infty$. Because $r(t) \in \mathcal{L}_\infty$, we can use Eq. (39) to show that $e(t)$ and $\dot{e}(t) \in \mathcal{L}_\infty$ (Ref. 21). Standard signal chasing arguments can now be employed to show that all other signals remain bounded. From Eqs. (51) and (52), we know that $r(t) \in \mathcal{L}_2$, whereas from Eq. (45) it is easy to show that $\dot{r}(t) \in \mathcal{L}_\infty$. We can now utilize Barbalat's lemma (see Refs. 7 and 21) to prove that

$$\lim_{t\to\infty} r(t) = 0 \tag{53}$$

Given the result of Eq. (53), we can use Eq. (39) to obtain the first result of Eq. (46) and also show that $\lim_{t\to\infty}\dot{e}(t) = 0$ (Ref. 21). Hence, from Eq. (25) and that P has full rank, we can prove the second result of Eq. (46). [This result is roughly speaking based on that Eq. (39) can be written as $\dot{e} = -\alpha e + r$, which is a stable linear system with input $r(t)$ and output $e(t)$. Thus, if $\lim_{t\to\infty} r(t) = 0$, then $\lim_{t\to\infty} e(t) = 0$.] □

IV. Adaptive Output Feedback Controller

In this section, we redesign the adaptive controller under the constraint that velocity measurements are not available. To facilitate the subsequent stability analysis, we define an auxiliary error signal, denoted by $\eta(t) \in \Re^3$, as follows:

$$\eta \triangleq \dot{e} + e + e_f \tag{54}$$

where $e_f(t) \in \Re^3$ is a filter signal that is yet to be designed.

A. Control Torque Input Design

Motivated by the desire to design a control torque input that is independent of velocity measurements, we construct the filter signal $e_f(t)$ as follows:

$$e_f = -ke + p \tag{55}$$

where $k \in \Re$ is a positive, constant control gain and $p(t) \in \Re^3$ is generated via the following dynamic expression:

$$\dot{p} = -(k+1)p + k^2 e + e/(1 - e^T e)^2, \qquad p(0) = ke(0) \tag{56}$$

Based on the subsequent stability analysis and structure of Eq. (30), we design the control input as follows:

$$u^* = -W_d\hat{\theta} + ke_f - e/(1 - e^T e)^2 \tag{57}$$

where $W_d(\omega_d, \dot{\omega}_d) \in \Re^{3\times 6}$ is a known regression matrix constructed according to the following parameterization:

$$W_d\theta = -J\dot{\omega}_d - \tfrac{1}{2}\omega_d^\times J\omega_d \tag{58}$$

where $\hat{\theta}(t)$ is the dynamic update law now designed as

$$\dot{\hat{\theta}} = \Gamma W_d^T\eta \tag{59}$$

and k is the same control gain defined in Eq. (55), which is selected as follows:

$$k = (1/j_1)(k_N + 1) \tag{60}$$

where j_1 was defined in Eq. (35) and $k_N \in \Re$ is an additional, positive, constant control gain.

Remark 5: The dynamic update law presented in Eq. (59) cannot be used for implementation because it depends on velocity. Rather, Eq. (59) is only used for stability analysis purposes. To illustrate that $\hat{\theta}(t)$ can be calculated using only measurable signals, we first rewrite Eq. (59) as the following integral expression:

$$\hat{\theta}(t) = \Gamma\int_0^t W_d^T[\omega_d(\sigma), \dot{\omega}_d(\sigma)][\dot{e}(\sigma) + e(\sigma) + e_f(\sigma)]\,d\sigma + \hat{\theta}(0) \tag{61}$$

After performing integration by parts, Eq. (61) can be written in the following velocity-independent form:

$$\hat{\theta}(t) = \Gamma\int_0^t \Big\{W_d^T(\cdot)[e(\sigma) + e_f(\sigma)]$$
$$- \dot{W}_d^T(\cdot)e(\sigma)\Big\}\,d\sigma + \Gamma W_d^T e + \hat{\theta}(0) \tag{62}$$

To determine the dynamics for $e_f(t)$, we take the time derivative of Eq. (55) and then substitute Eq. (56) into the resulting expression to obtain

$$\dot{e}_f = -k\dot{e} - (k+1)p + k^2 e + e/(1 - e^T e)^2 \tag{63}$$

After rearranging Eq. (55), we can substitute for $p(t)$ in Eq. (63) and then simplify the resulting expression to obtain the following:

$$\dot{e}_f = -k\eta - e_f + e/(1 - e^T e)^2 \tag{64}$$

where Eq. (54) was utilized. To develop the open-loop expression for $\eta(t)$, we take the time derivative of Eq. (54), premultiply the resulting expression by J^*, and then substitute Eq. (30) for $J^*\ddot{e}$ to obtain

$$J^*\dot{\eta} = u^* - C^*\dot{e} - N^* + J^*\dot{e} + J^*\dot{e}_f \tag{65}$$

After utilizing Eqs. (54) and (64), we can rewrite Eq. (65) as follows:

$$J^*\dot{\eta} = u^* + (1-k)J^*\eta - 2J^*e_f - J^*e - C^*\eta$$
$$+ J^*[e/(1-e^Te)^2] + C^*e_f + C^*e - N^* \tag{66}$$

After adding and subtracting $W_d\theta$ to the right side of Eq. (66) and substituting Eq. (57) for $\boldsymbol{u}^*(t)$, we can obtain the following expression for the closed-loop error system for $\eta(t)$:

$$J^*\dot{\eta} = \chi + W_d\tilde{\theta} + ke_f - e/(1-e^Te)^2 - kJ^*\eta - C^*\eta \tag{67}$$

where $\tilde{\theta}(t)$ was defined in Eq. (37) and the auxiliary signal $\chi(e, e_0, e_f, \eta, \omega_d, \dot{\omega}_d) \in \Re^3$ is defined as follows:

$$\chi \triangleq C^*(e_f + e) + J^*[\eta - e + e/(1-e^Te)^2]$$
$$- 2J^*e_f - W_d\theta - N^* \tag{68}$$

Remark 6: To facilitate the subsequent stability analysis, we utilize Eqs. (68) and (29) to construct the following auxiliary variable:

$$\bar{\chi} \triangleq P^{-T}\chi \tag{69}$$

Based on the assumptions on the boundedness of the desired trajectory and the structure of Eq. (69), we can show that $\bar{\chi}$ can be upper bounded as follows (see Appendix C):

$$\bar{\chi} \leq \rho(\|z\|)\|z\| \tag{70}$$

where $\rho(\cdot)$ is a positive, nondecreasing function and the auxiliary signals $z(t) \in \Re^9$ and $y(t) \in \Re^3$ are now defined as follows:

$$z \triangleq \left[e^T\big/\sqrt{1-e^Te} \quad e_f^T \quad y^T\right]^T \tag{71}$$

$$y \triangleq P\eta \tag{72}$$

B. Stability Analysis

Theorem 2: Given closed-loop error systems of Eqs. (54), (59), (64), and (67), the adaptive controller of Eqs. (55–57) and (62) ensures asymptotic attitude tracking in the sense that

$$\lim_{t\to\infty} e(t) = 0, \qquad \lim_{t\to\infty} \tilde{\omega}(t) = 0 \tag{73}$$

provided that the initial conditions are selected such that Eq. (47) is satisfied, and the control gain k_N introduced in Eq. (60) is selected according to the following inequality:

$$k_N > \rho^2\left[\sqrt{\lambda_2/\lambda_1}\|x(0)\|\right] \tag{74}$$

where $\rho(\cdot)$ was defined in Eq. (70) and $x(t) \in \Re^{12}$ is defined as follows:

$$x \triangleq [z^T \quad \tilde{\theta}^T]^T \tag{75}$$

Also λ_1 and λ_2 are positive constants defined as

$$\lambda_1 \triangleq \tfrac{1}{2}\min\left\{1, j_1, \lambda_{\min}\{\Gamma^{-1}\}\right\} \qquad \lambda_2 \triangleq \tfrac{1}{2}\max\left\{1, j_2, \lambda_{\max}\{\Gamma^{-1}\}\right\} \tag{76}$$

and $\lambda_{\min}\{\cdot\}$ and $\lambda_{\max}\{\cdot\}$ represent the minimum and maximum eigenvalue of a matrix, respectively.

Proof: To prove the Theorem 2, we define the nonnegative function

$$V \triangleq \tfrac{1}{2}[e^Te/(1-e^Te)] + \tfrac{1}{2}e_f^Te_f + \tfrac{1}{2}y^TJy + \tfrac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \tag{77}$$

Based upon the structure of Eq. (77) and Property 2, we can lower and upper bound $V(t)$ as follows:

$$\lambda_1\|x\|^2 \leq V \leq \lambda_2\|x\|^2 \tag{78}$$

where $x(t)$ was defined in Eq. (75). After taking the time derivative of Eq. (77), substituting Eq. (72) for $y(t)$, using the definition of J^* from Eq. (28), substituting Eq. (67), and then simplifying the resulting expression, we obtain

$$\dot{V} = e^T\dot{e}/(1-e^Te)^2 + e_f^T\dot{e}_f + \tfrac{1}{2}\eta^T\dot{J}^*\eta + \tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}}$$
$$+ \eta^T\left[\chi + W_d\tilde{\theta} + ke_f - e/(1-e^Te)^2 - kJ^*\eta - C^*\eta\right] \tag{79}$$

After utilizing Eqs. (54), (64), (69), (72), and Property 1, we can rewrite (79) as follows:

$$\dot{V} = -[e^Te/(1-e^Te)^2] - e_f^Te_f + y^T\bar{\chi}$$
$$- \eta^TkJ^*\eta + \tilde{\theta}^T\left(W_d^T\eta + \Gamma^{-1}\dot{\tilde{\theta}}\right) \tag{80}$$

After utilizing Eqs. (28), (59), and (72), expression (80) can be upper bounded as follows:

$$\dot{V} \leq -[e^Te/(1-e^Te)^2] - e_f^Te_f - kj_1\|y\|^2 + \|y\|\|\bar{\chi}\| \tag{81}$$

After substituting Eqs. (60), (70), and (71) into Eq. (81), we obtain the following expression:

$$\dot{V} \leq -\|z\|^2 + \left[\|y\|\,\|z\|\rho(\|z\|) - k_N\|y\|^2\right] \tag{82}$$

After applying the nonlinear damping tool[21] to the bracketed term in Eq. (82), we obtain

$$\dot{V} \leq -\left[1 - \rho^2(\|z\|)/k_N\right]\|z\|^2 \tag{83}$$

Note that from Eq. (83) we can write

$$\dot{V} \leq -\beta\|z\|^2 \qquad \text{for} \qquad k_N > \rho^2(\|x\|) \tag{84}$$

where β is some positive constant and we have used that $\|x\| \geq \|z\|$, as indicated by Eq. (75). Upon utilization of Eq. (78), we can develop a sufficient condition for Eq. (84) as follows:

$$\dot{V} \leq -\beta\|z\|^2 \qquad \text{for} \qquad k_N > \rho^2\left[\sqrt{V(t)/\lambda_1}\right] \tag{85}$$

From Eqs. (78) and (85), we can see that $V(t)$ is nonnegative and $\dot{V}(t) \leq 0$; hence, we can conclude that

$$0 \leq V(t) \leq V(0) < \infty \tag{86}$$

We now use Eqs. (78) and (85) to develop a sufficient condition for Eq. (85) as follows:

$$\dot{V} \leq -\beta\|z\|^2 \qquad \text{for} \qquad k_N > \rho^2\left(\sqrt{\lambda_2/\lambda_1}\|x(0)\|\right) \tag{87}$$

Given Eqs. (22), (23), and (47), it is clear that $\|e(0)\| < 1$. From Eqs. (86) and (51), it is straightforward to see from Eq. (77) that $y(t)$, $\hat{\theta}(t)$, $e_f(t) \in \mathcal{L}_\infty$, and $\|e(t)\| < 1$ for all time. Because $\|e(t)\| < 1$ for all time, we can conclude from Eq. (22) that $e_0(t) \neq 0$ for all time; hence, we know from Eq. (36) that P defined in Eq. (29) has full rank for all time. Because $y(t) \in \mathcal{L}_\infty$ and P has full rank for all time, we can use Eq. (72) to show that $\eta(t) \in \mathcal{L}_\infty$. We can now utilize this information and Eq. (54) to show that $\dot{e}(t) \in \mathcal{L}_\infty$. Standard signal chasing arguments can now be employed to show that all other signals remain bounded.

From Eqs. (84) and (86), we know that $z(t) \in \mathcal{L}_2$. Given that all signals are bounded and the fact that $\|e(t)\| < 1$ for all time, we can use Eqs. (71), (54), (64), and (67) to show that $\dot{z}(t) \in \mathcal{L}_\infty$. We can now utilize Barbalat's lemma to prove that

$$\lim_{t\to\infty} z(t) = 0 \qquad \text{for} \qquad k_N > \rho^2\left[\sqrt{\lambda_2/\lambda_1}\|x(0)\|\right] \tag{88}$$

Given the result of Eq. (88), we can use Eqs. (25), (54), (71), (72), and that P has full rank to obtain Eq. (73). □

V. Simulation Results

We now present a numerical simulation to validate the adaptive output feedback controller developed in the preceding section. The inertia matrix was selected as follows[2]:

$$J = \begin{bmatrix} 20 & 1.2 & 0.9 \\ 1.2 & 17 & 1.4 \\ 0.9 & 1.4 & 15 \end{bmatrix} \tag{89}$$

whereas the initial attitude of the spacecraft was set to

$$q_0(0) = 0.9486 \qquad q(0) = [0.1826 \quad 0.1826 \quad 0.1826]^T \tag{90}$$

The desired attitude trajectory was selected to be a smooth rotation about each axis as shown by Fig. 1 with the corresponding desired angular velocity as follows:

$$\omega_d(t) = \begin{Bmatrix} 0.3\cos(t)\left(1 - e^{-0.01t^2}\right) + [0.08\pi + 0.006\sin(t)]te^{-0.01t^2} \\ 0.3\cos(t)\left(1 - e^{-0.01t^2}\right) + [0.08\pi + 0.006\sin(t)]te^{-0.01t^2} \\ 0.3\cos(t)\left(1 - e^{-0.01t^2}\right) + [0.08\pi + 0.006\sin(t)]te^{-0.01t^2} \end{Bmatrix} \text{rad/s} \tag{91}$$

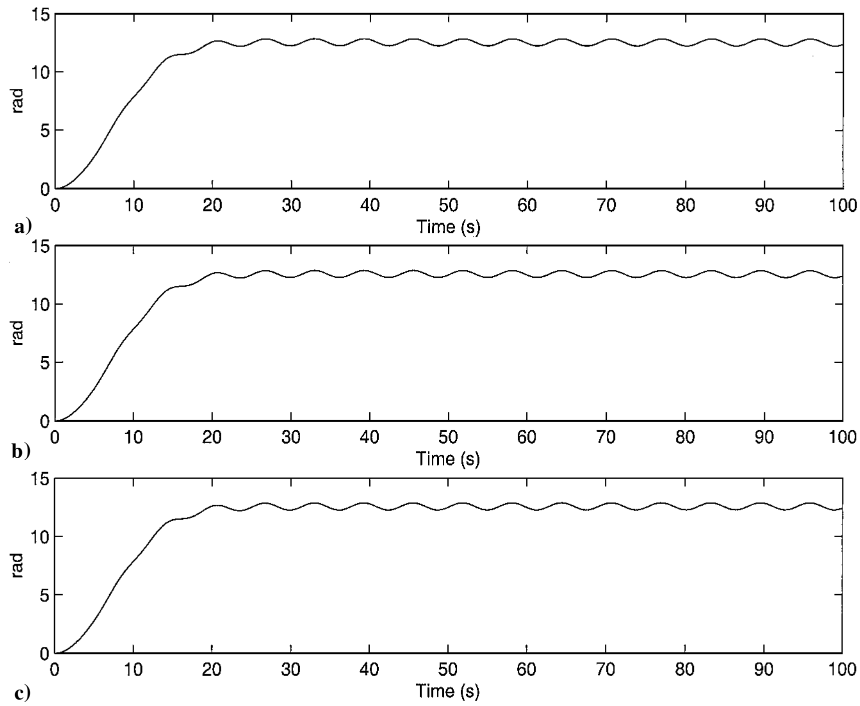

Fig. 1 Desired angular position about a) *x* axis, b) *y* axis, and c) *z* axis.

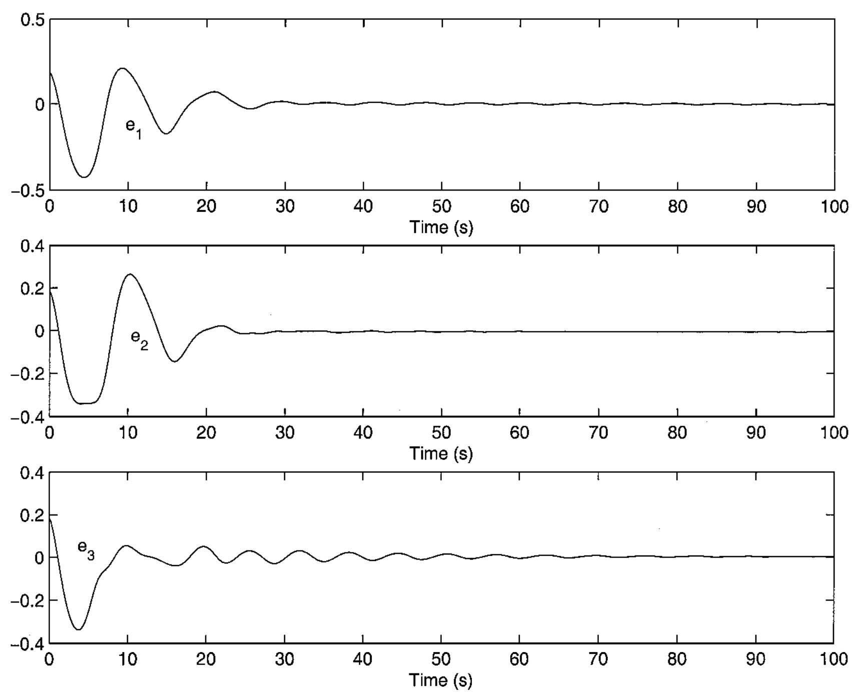

Fig. 2 Position tracking error *e*(*t*).

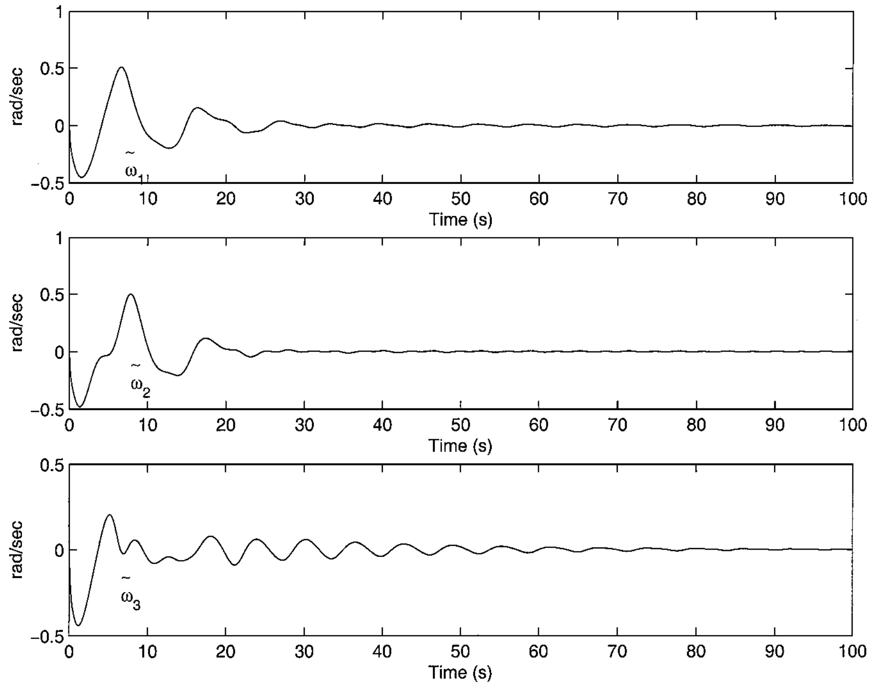

Fig. 3 Angular velocity tracking error $\tilde{\omega}(t)$.

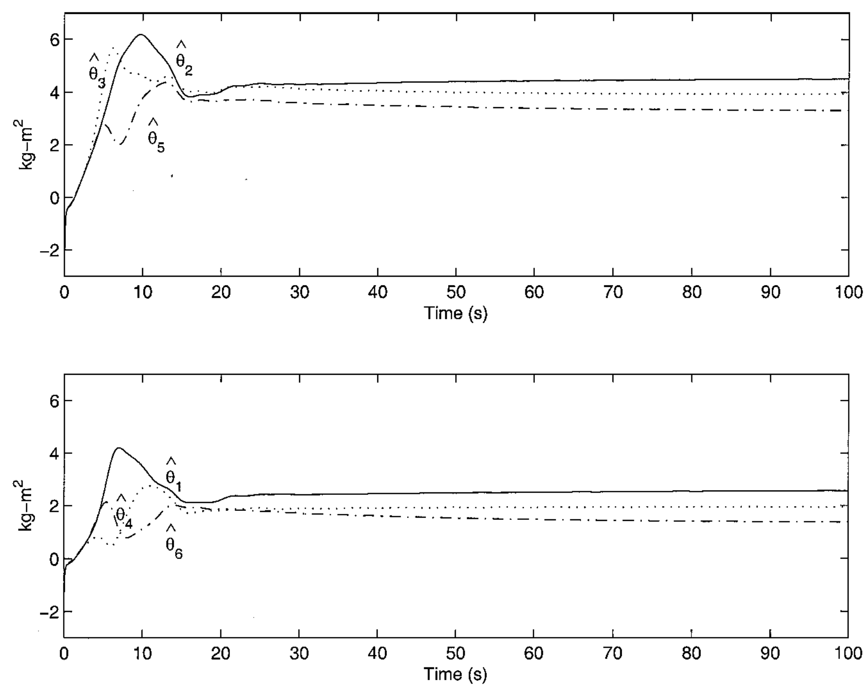

Fig. 4 Parameter estimates $\hat{\theta}(t)$.

The desired quaternion is related to the desired angular velocity of Eq. (91) through the kinematic equations (10) and (11) with the following initial conditions:

$$q_{0d}(0) = 1 \qquad q_d(0) = [0 \quad 0 \quad 0]^T \tag{92}$$

where Eq. (8) has been satisfied. The control gain in Eqs. (55–57) and the adaptation gain matrix in Eq. (62) were tuned by trial and error until the best tracking performance was achieved. Their values are

$$k = 10 \qquad \Gamma = \mathrm{diag}\{30, 30, 30, 30, 30, 30\} \tag{93}$$

Furthermore, the parameter estimates were initialized to zero, that is, $\hat{\theta}(0) = 0$. From Figs. 2 and 3, we can see that the tracking control objective has been achieved. The dynamic estimates of the inertia matrix are shown in Fig. 4 The three control torques can be found in Fig. 5.

Remark 7: We have selected a desired angular position trajectory to show that the controller avoids all potential singularities that are associated with the three parameter orientations that are discussed in the Introduction. To do this we have selected the desired angular position about the x, y, and z axis to be

$$\phi_d(t) = \begin{Bmatrix} [4\pi + 0.3\sin(t)]\left(1 - e^{-0.01t^2}\right) \\ [4\pi + 0.3\sin(t)]\left(1 - e^{-0.01t^2}\right) \\ [4\pi + 0.3\sin(t)]\left(1 - e^{-0.01t^2}\right) \end{Bmatrix} \text{rad} \tag{94}$$

This trajectory is a smooth rotation through two complete revolutions with a small sinusoid superimposed on it. The desired angular velocity found in Eq. (91) is found by taking the time derivative of Eq. (94).

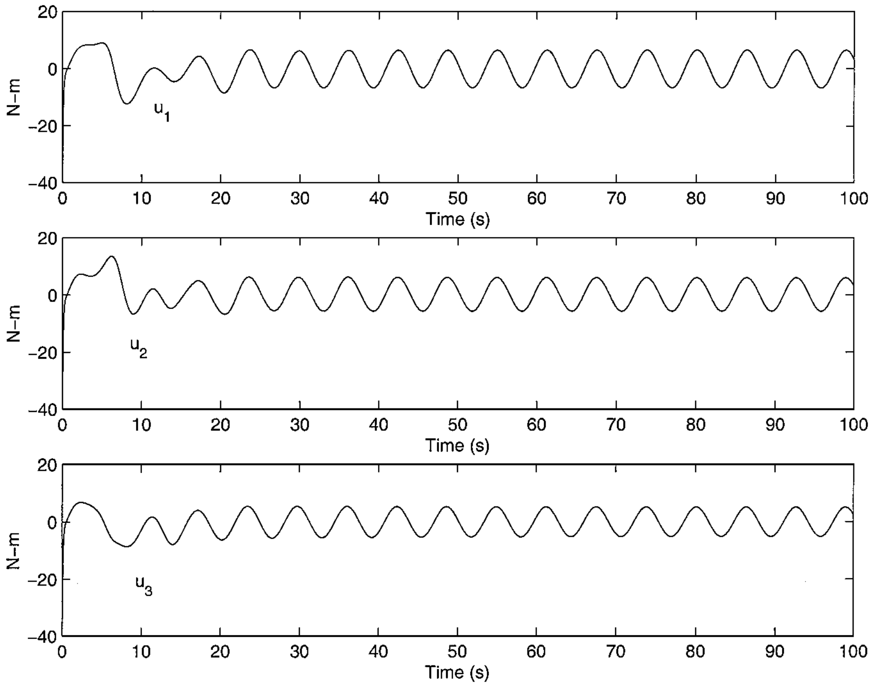

Fig. 5 Control torques $u(t)$.

VI. Conclusions

We have presented two adaptive controllers that address the attitude tracking problem for rigid spacecraft based on the unit quaternion, kinematic representation. The first controller is a full-state feedback controller that adapts for the unknown spacecraft inertia matrix and achieves asymptotic attitude tracking. The adaptive controller is then redesigned to eliminate the need for angular velocity measurements and still obtains asymptotic attitude tracking. Because the proposed controller is based on Lyapunov stability analysis, several extensions to the proposed work are straightforward. For example, one could easily use the full-state feedback controller structure to develop variable structure or high-gain/high-frequency robust controllers that compensate for parameteric uncertainty and additive bounded disturbances, while producing exponential tracking and uniform ultimate boundedness tracking, respectively. In addition, one could easily use the output feedback controller structure to develop model-based or high-gain robust controllers that compensate for parameteric uncertainty and additive bounded disturbances while producing exponential tracking and uniform ultimate boundedness tracking, respectively. Future plans for this research will include experimental verification on a gyroscopic testbed.

Appendix A: Transformed Error System Development

After substituting Eqs. (18), (25), and (29) into Eq. (27), we obtain

$$J^*\ddot{e} = J^*\dot{P}^{-1}P\dot{e} + \tfrac{1}{2}P^T\Big[-(\tilde{\omega}+\tilde{R}\omega_d)^\times J(\tilde{\omega}+\tilde{R}\omega_d) + J\big(\tilde{\omega}^\times\tilde{R}\omega_d - \tilde{R}\dot{\omega}_d\big) + u\Big] \tag{A1}$$

which can be rearranged to yield

$$J^*\ddot{e} - J^*\dot{P}^{-1}P\dot{e} + \tfrac{1}{2}P^T(\tilde{\omega}+\tilde{R}\omega_d)^\times J(\tilde{\omega}+\tilde{R}\omega_d) - \tfrac{1}{2}P^TJ\big(\tilde{\omega}^\times\tilde{R}\omega_d - \tilde{R}\dot{\omega}_d\big) = \tfrac{1}{2}P^Tu \tag{A2}$$

We define the auxiliary signal $\Omega_1(e, e_0, \omega, \omega_d) \in \Re^3$ as follows:

$$\Omega_1 \triangleq \tfrac{1}{2}P^T\big(\tilde{\omega}+\tilde{R}\omega_d\big)^\times J(\tilde{\omega}+\tilde{R}\omega_d) \tag{A3}$$

which can be expanded to yield

$$\Omega_1 = \tfrac{1}{2}P^T(\tilde{\omega}^\times J\tilde{\omega}) + \tfrac{1}{2}P^T\big(\tilde{\omega}^\times J\tilde{R}\omega_d\big) + \tfrac{1}{2}P^T\big[(\tilde{R}\omega_d)^\times J\tilde{\omega}\big] + \tfrac{1}{2}P^T\big[(\tilde{R}\omega_d)^\times J\tilde{R}\omega_d\big] \tag{A4}$$

Now, we will define the auxiliary signal $\Omega_2(e, e_0, \omega) \in \Re^3$ as follows:

$$\Omega_2 \triangleq \tfrac{1}{2}P^T(\tilde{\omega}^\times J\tilde{\omega}) \tag{A5}$$

After substituting for $\tilde{\omega}(t)$ in Eq. (A5) by manipulating Eq. (25) and then substituting Eq. (29) for $T^{-1}(t)$, we obtain

$$\Omega_2 = \tfrac{1}{2}P^T[(2P\dot{e})^\times 2JP\dot{e}] \tag{A6}$$

which can be simplified to

$$\Omega_2 = 2P^T(P\dot{e})^\times JP\dot{e} = -2P^T(JP\dot{e})^\times P\dot{e} \tag{A7}$$

where Eq. (4) has been used. After substituting the equalities of Eqs. (A4) and (A6) into Eq. (A2), we can now write the model in the form found in Eq. (30).

Appendix B: Skew-Symmetry Property

To prove Eq. (34), we first take the time derivative of Eq. (28) and then utilize Eq. (32) to obtain the following expression:

$$\begin{aligned} x^T(\dot{J}^* - 2C^*)x &= x^T(\dot{P}^TJP + P^TJ\dot{P})x \\ &\quad + 2x^T(J^*\dot{P}^{-1}P + 2P^T(JP\dot{e})^\times P)x \qquad \forall x \in \Re^3 \\ &= 2x^T(P^TJ\dot{P} + P^TJP\dot{P}^{-1}P)x \end{aligned} \tag{B1}$$

where we have used that $x^TP^T(JP\dot{e})^\times Px = 0$. After making use of

$$\dot{P}^{-1} = -P^{-1}\dot{P}P^{-1} \tag{B2}$$

Eq. (B1) can be rewritten as

$$x^T(\dot{J}^* - 2C^*)x = 2x^T(P^TJ\dot{P} - P^TJ\dot{P})x = 0 \tag{B3}$$

Appendix C: Bound on $\bar{\chi}$

After making use of Eqs. (58), (68), and (69), we can express $\bar{\chi}$ as follows:

$$\bar{\chi} = P^{-T}C^*(e_f + e) + P^{-T}J^*[\eta - e + e/(1 - e^Te)^2] - 2P^{-T}J^*e_f + P^{-T}J\dot{\omega}_d + \tfrac{1}{2}P^{-T}\omega_d^\times J\omega_d - P^{-T}N^* \tag{C1}$$

The expression of Eq. (C1) can be upper bounded as follows:

$$\bar{\chi} \le \|P^{-T}C^*\|(\|e_f\| + \|e\|) + \|P^{-T}J^*\|(\|\eta\| + \|e\| + \|e/(1 - e^Te)^2\|) + 2\|P^{-T}J^*\|\|e_f\| + \|\psi\| \tag{C2}$$

where $\psi(e, e_0, e_f, \eta, \omega_d, \dot{\omega}_d) \in \Re^3$ is defined as

$$\psi \triangleq P^{-T} J \dot{\omega}_d + \tfrac{1}{2} P^{-T} \omega_d^{\times} J \omega_d - P^{-T} N^* \tag{C3}$$

The expression in Eq. (C3) can be manipulated using Eq. (33) to obtain

$$\psi = \left[(J\tilde{R}\omega_d)^{\times} - (\tilde{R}\omega_d)^{\times} J - J(\tilde{R}\omega_d)^{\times}\right] P(\eta - e - e_f) + \left(P^{-T} J - \tfrac{1}{2} J\tilde{R}\right)\dot{\omega}_d + \tfrac{1}{2}\left\{P^{-T}\omega_d^{\times} J\omega_d - \left[(\tilde{R}\omega_d)^{\times} J\tilde{R}\omega_d\right]\right\} \tag{C4}$$

After making use of Eqs. (13), (22), (26), and (29), we can simplify Eq. (C4) to obtain the following bound for $\|\psi\|$:

$$\|\psi\| \le \rho_1(e)(\|\eta\| + \|e\| + \|e_f\|) + \rho_2(e)\left(\left\| e\big/\sqrt{1 - e^T e}\right\|\right) \tag{C5}$$

where $\rho_1(\cdot)$ and $\rho_2(\cdot)$ are some positive, nondecreasing functions. Based on the definition of Eqs. (22), (28), (29), (32), (71), and (C5), the expression in Eq. (C1) can be simplified to yield the result found in Eq. (70).

Acknowledgments

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